Tenth International Conference on Managing Fatigue: Abstract for Review

Discovery that a person's level of drowsiness appears to evolves in time according to a Geometric Brownian Motion (GBM) random process model

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Problem

Drowsiness causes about 30% of highway accidents. It is thus paramount to monitor the level of drowsiness (LoD) of a driver and to act appropriately. We focus here on systems that monitor the physiological state of the subject, e.g. by using images of an eye. All systems that we know of can establish a present LoD based on such data obtained up to the present time. But, if the LoD at the present time reaches a critical level, it may be too late to save a driver's life. Therefore, it is critical for drowsiness monitoring systems to also be able to predict future LoD values - at least a few (tens of) seconds ahead - based on data recorded up to the present time.

Method

A conventional strategy for predicting future values of a signal is to describe this signal via a model. Since the evolution of the LoD is inherently random, one must treat each real-life "LoD signal" as a realization of a random process (RP). The goal then becomes to identify RP models that are appropriate for such LoD signals. We started our investigation by considering Autoregressive (AR) and Autoregressive Integrated Moving Average (ARIMA) models, which already proved quite useful. In pursuing our investigation, we discovered that the RP process model called **Geometric Brownian Motion** (**GBM**) might be very useful to model LoD signals and predict future values thereof. The goal of this paper is to show that real-life LoD signals are indeed well modeled by GBM RP models, or GBMs, for short.

A stochastic process $X(t)$ is said to follow a GBM if it satisfies the stochastic differential equation

$$dX(t) = \mu X(t)dt + \sigma X(t)dW(t)\,, \qquad (1)$$

where $W(t)$ is a Weiner (random) process or **Brownian Motion (BM)**, and μ (the percentage drift) and σ (the percentage volatility) are constant. Intuitively, $\mu X(t)dt$ controls the *trend* of the trajectory, and $\sigma X(t)dW(t)$ the *random noise* effect in it. For an arbitrary initial value $X(0)$, the above equation is known to have the analytical solution

$$X(t) = X(0)\exp\left(\left(\mu - \frac{\sigma^2}{2}\right)t - \sigma W(t)\right). \qquad (2)$$

In contrast with many other conventional RP models (such as Autoregressive (AR), Moving Average (MA), Autoregressive Moving Average (ARMA), and Autoregressive Integrated Moving Average (ARIMA)), there is a simple and accurate procedure to check whether the RP follows a GBM or not, and to also find its corresponding parameters. In short, a RP follows a GBM if the logarithm of the ratios of successive values constitute an independent and identically distributed (i.i.d.) Gaussian RP.

Results

The LoD signals used here were produced using a drowsiness monitoring system built in our group, and consisting of a camera mounted on a pair of eyeglasses. This system continuously takes images of an eye, and automatically produces a validated LoD signal. In this study, we used the LoD signals from 13 healthy subjects who performed psychomotor vigilance tasks (PVTs) at three different states of sleep deprivation (over three days), i.e. 3x13 = 39 signals, each with 42 samples computed every 20 sec. For each of these real-life LoD signals, we examined whether or not they could be viewed as being realizations of GBMs.

The conventional procedure to determine whether a RP is GBM has two successive steps: (1) verifying that the logarithms of the ratios of successive values are normally distributed; (2) verifying that the ratios of successive values are uncorrelated (in time). For the first step, we applied, to each signal, established graphical methods, i.e. the quantile-quantile (QQ) plot and the histogram. The mere inspection of the plots below shows that the corresponding signal meets the first requirement.

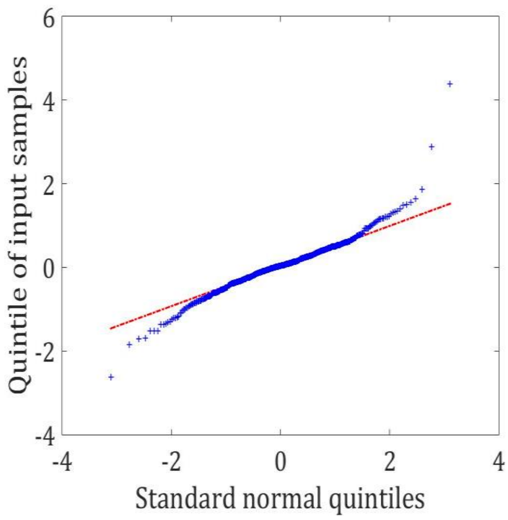

Figure 1: QQ plot of Log-Ratios of one particular LoD signal.

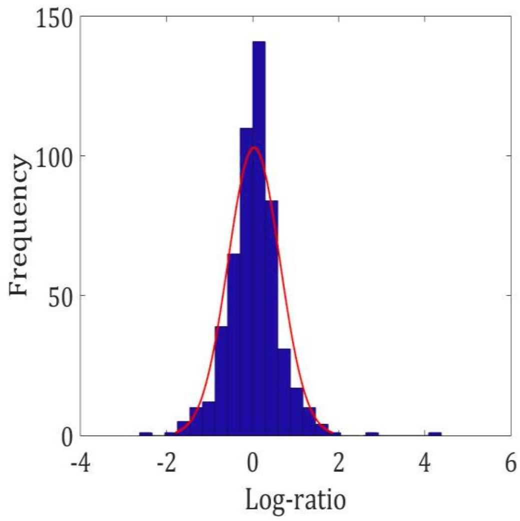

Figure 2: Histogram of Log-Ratios of same particular LoD signal.

For the second step, we looked at the scatter plot of Log-Ratios versus time of each signal to see whether there was any (time) correlation between the logarithms of the ratios of successive values.

All 39 available, real-life LoD signals successfully passed all the above tests and were thus declared to be realizations of GBM RPs. Furthermore, we were able to determine the values of the parameters of each of them. This also means that we would be able to predict the future values of each of these signals.

Discussion

The GBM RP has been successfully used in physics and finance, but is little known and virtually unused in other fields. In finance, it is frequently used as a model for diverse quantities such as stock prices, natural resource prices, and the growth in demand for products or services. Robert C. Merton and Myron S. Scholes have, in collaboration with the late Fischer Black, developed a pioneering model for the valuation of stocks based on the GBM RP model. Their model - the Black–Scholes options pricing model - led to the Nobel Prize in Economics for Merton and Scholes in 1997. Our quest for an appropriate, useful RP model for LoD signals led us at some point to the GBM RP model. The preliminary results described here indicate that the GBM might be the killer model for LoD signals! We have also shown that other, similar, physiological signals are also GBM. This paper may, in fact, be the first application ever of a GBM RP model to a biological signal.

Summary

We performed three different statistical tests on the LoD signals from 13 healthy subjects who performed PVTs at three different states of sleep deprivation (over three days) to validate our hypothesis that LoD signals follow GBM RP models. The preliminary results described here strongly suggest that the LoD of a person evolves according to a GBM. The discovery that LoD signals (as well as other related signals) are GBM opens up new avenues of research in drowsiness monitoring and in the prediction of the future values of such signals. To the best of our knowledge, this may also be the first time that a GBM - mainly used in finance - is envisioned to model a biological signal.