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The Science of Performance at Work




Accounting for Sleep Inertia in the Differential Equation Framework of a Biomathematical Model of Fatigue

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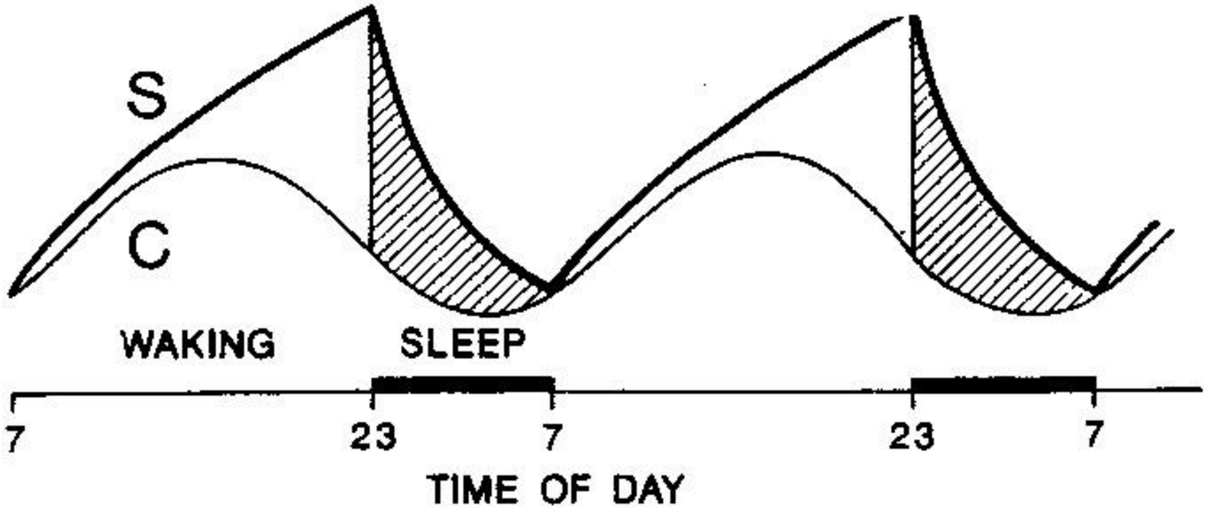
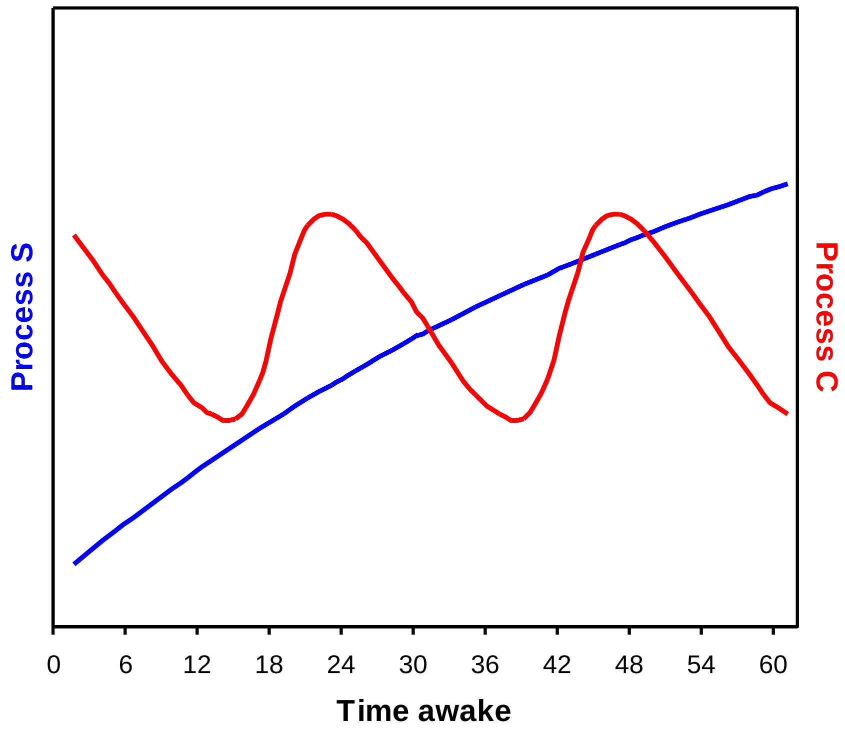
Two-Process Model of Sleep

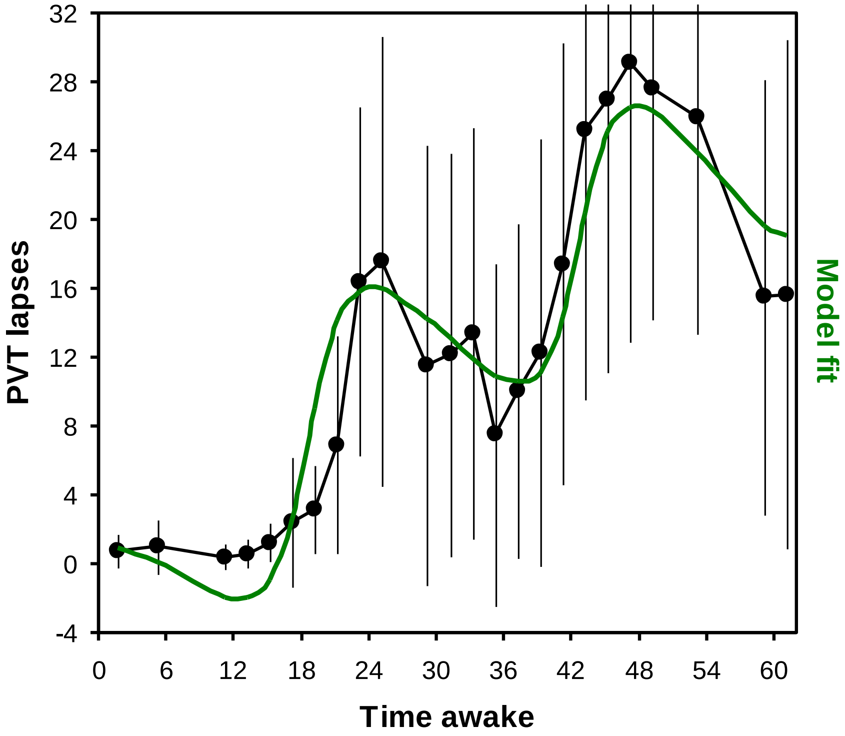
- Sleep and wakefulness governed by *biological* mechanisms
- Two-process model of sleep regulation:
  - *Homeostatic process*: builds up pressure for sleep during wakefulness and dissipates pressure during sleep
  - *Circadian process*: modulates sleep pressure as a function of time of day



Borbély AA (1982). A two process model of sleep regulation. Hum Neurobiol 1: 195-204.

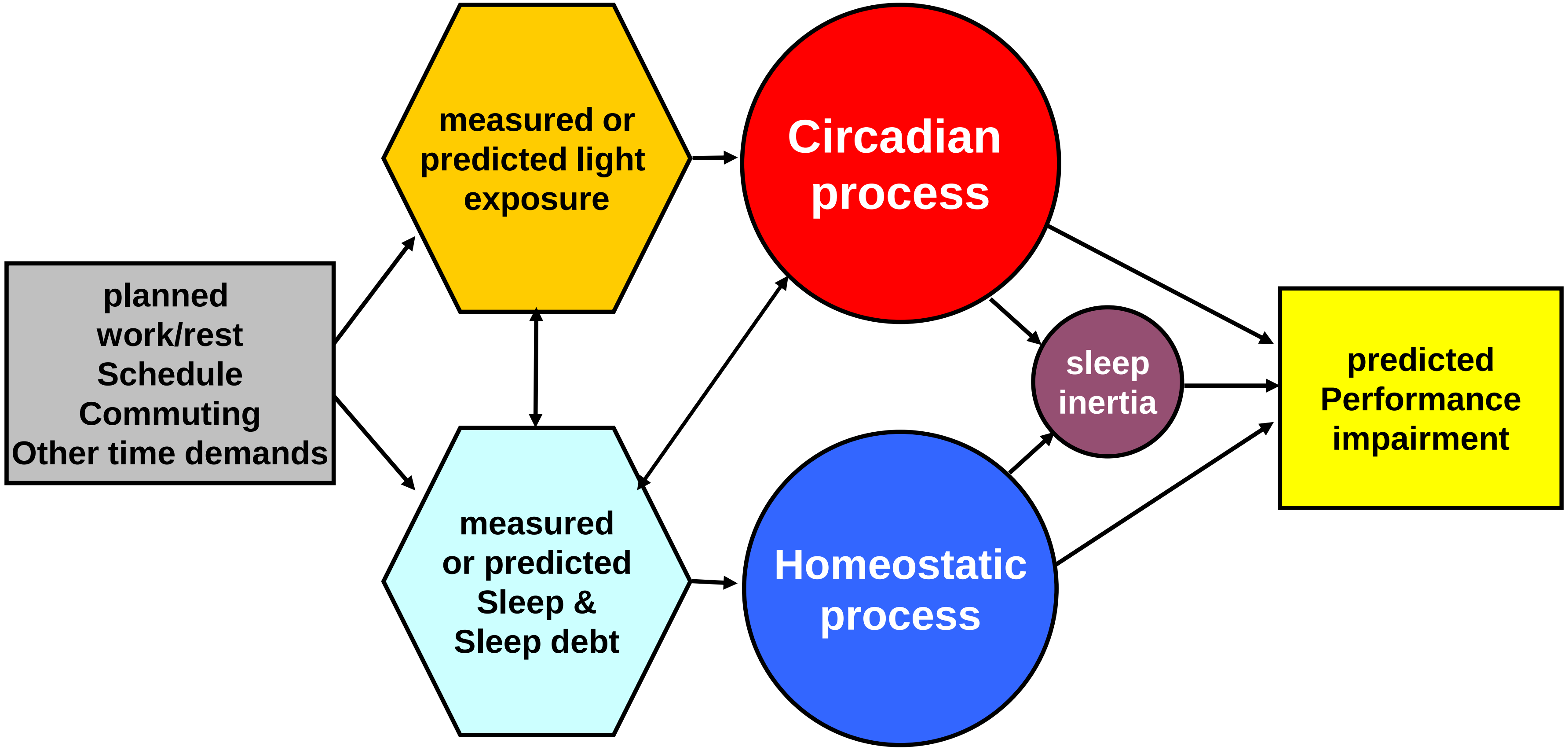
Two Basic Processes for Modeling Performance: Homeostatic Process and Circadian Process





Borbély AA (1982). A two process model of sleep regulation. Hum Neurobiol 1: 195-204.
Van Dongen HPA, Belenky G (2009). Individual differences in vulnerability to sleep loss in the work environment. Ind Health 47: 518-526.

Simplified Schematic of Performance Prediction Models



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Biomathematical modeling of sleep/wake homeostasis and allostasis

McCauley et al. (2009/2013) introduced a predictive model by applying a system of first-order ordinary differential equations (ODE's):

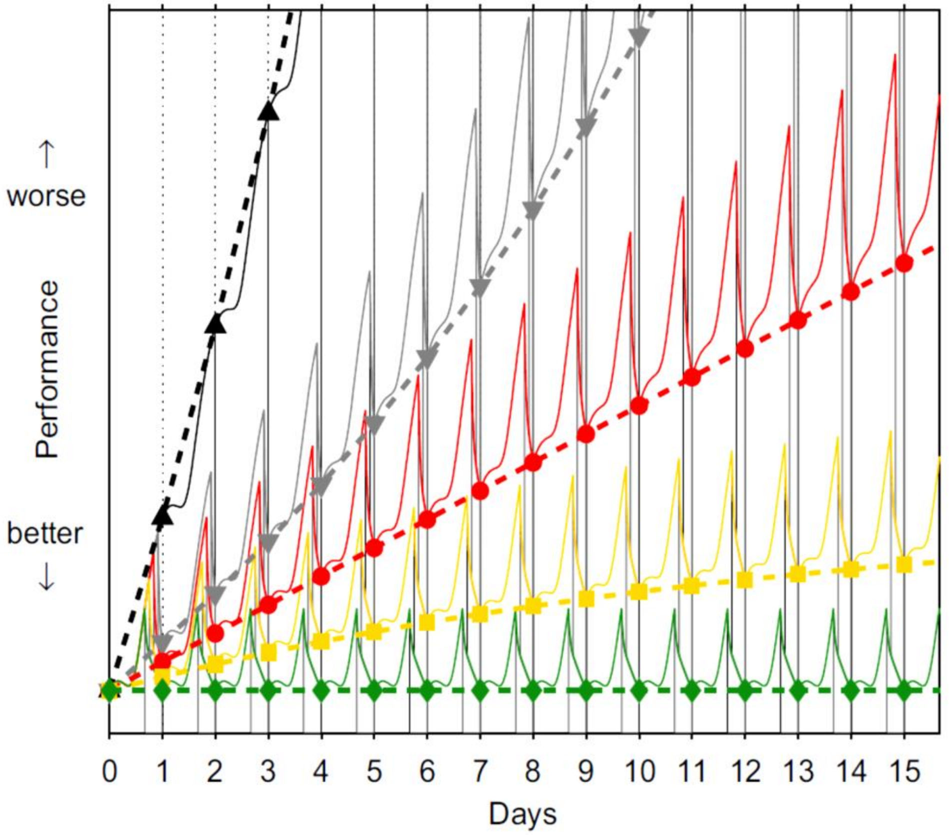
$$\frac{d\boldsymbol{p}(\boldsymbol{t})}{d\boldsymbol{t}} = \boldsymbol{\alpha}_{\boldsymbol{W}}[\mathbf{p}(\mathbf{t}) + \boldsymbol{\beta}_{\boldsymbol{W}}\boldsymbol{u}(\boldsymbol{t})] + \boldsymbol{\kappa}(\boldsymbol{t})[\boldsymbol{c}(\boldsymbol{t}) + \boldsymbol{\mu}_{\boldsymbol{W}}],$$

$$\frac{d\boldsymbol{u}(\boldsymbol{t})}{d\boldsymbol{t}} = \boldsymbol{\eta}_{\boldsymbol{W}}\boldsymbol{u}(\boldsymbol{t}),$$

$$\frac{d\boldsymbol{\kappa}(\boldsymbol{t})}{d\boldsymbol{t}} = \boldsymbol{\lambda}_{\boldsymbol{W}}\boldsymbol{\kappa}(\boldsymbol{t})\left(\mathbf{1} - \frac{\boldsymbol{\kappa}(\boldsymbol{t})}{\boldsymbol{\xi}}\right),$$

where:

- $\mathbf{p}(\mathbf{t})$ is the performance prediction in terms of lapses,
- $\boldsymbol{u}(\boldsymbol{t})$ represents the allostatic process,
- $\boldsymbol{\kappa}(\boldsymbol{t})$ represents dynamic changes in circadian amplitude
- $\boldsymbol{c}(\boldsymbol{t})$ represents 24-hour circadian rhythm oscillator.



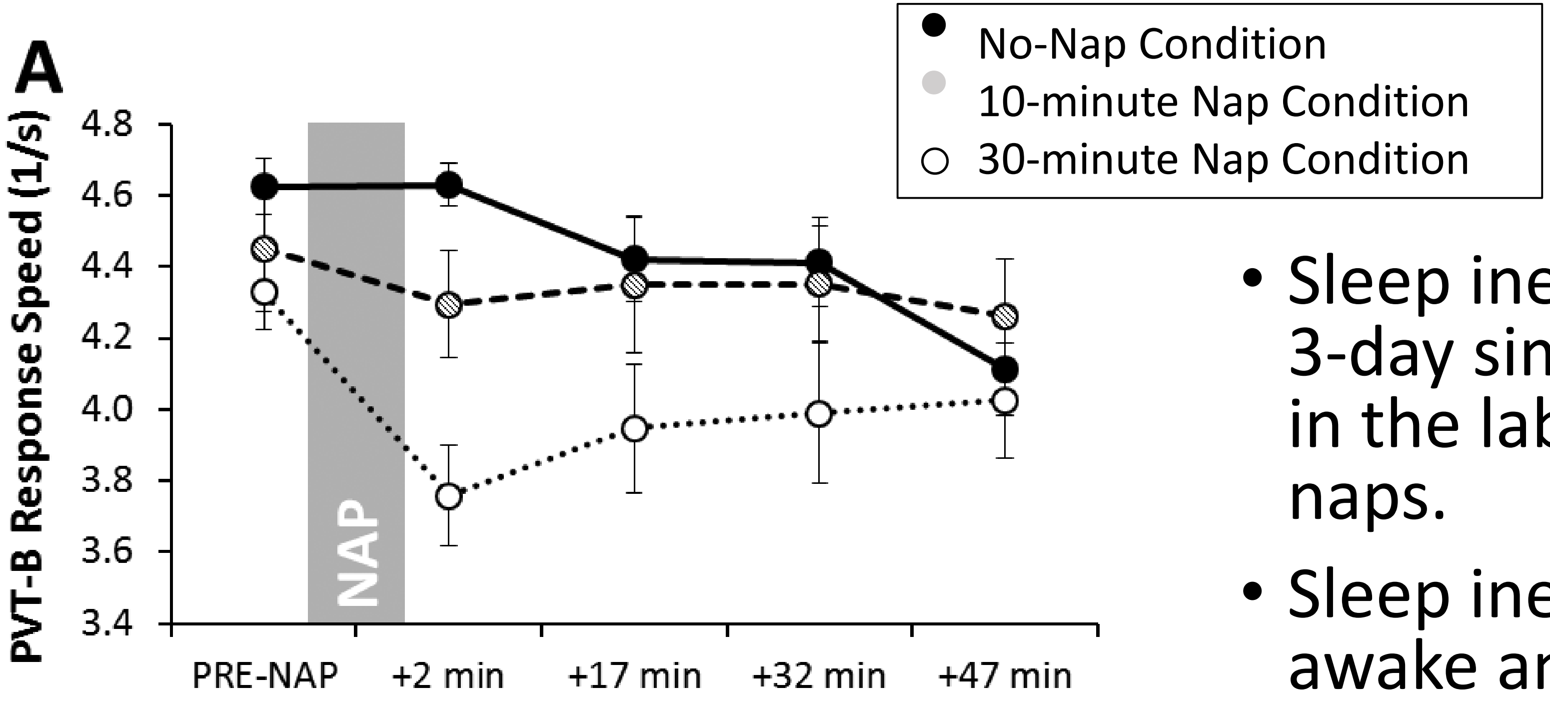
McCauley P, Kalachev LV, Smith AD, Belenky G, Dinges DF, Van Dongen HPA (2009). J Theor Biol 256: 227-239.
McCauley P, Kalachev LV, Mollicone DJ, Banks S, Dinges DF, Van Dongen HPA (2013). Sleep 36: 1987-1997.

Sleep Inertia

- *Sleep inertia*: refers to a period of reduced alertness, grogginess and impaired cognitive performance experienced immediately upon awakening
- Magnitude of effects dependent on multiple factors such as: sleep duration, sleep stage prior to awakening, prior sleep deprivation, time of day of awakening, etc.
- Sleep inertia effect declines upon awakening exponentially over time awake.

Tassi, P, Muzet, A (2000). Sleep Inertia. Sleep Medicine Reviews 4(4): 341-353.

Sleep Inertia and Performance



Hilditch, CJ, Centofanti, SA, Dorrian J, Banks, S (2016). A 30-minute, but not a 10-miute nighttime nap is associated with sleep inertia. Sleep 39(3): 675-685.

- Sleep inertia effects captured in a 3-day simulated night shift study in the laboratory using scheduled naps.
- Sleep inertia, in addition to time awake and time of day, impacts the magnitude and rate of change of performance upon awakening.

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Enhanced Model with Sleep Inertia

1. To include sleep inertia, an additional differential equation was formulated:

$$\frac{dx(t)}{dt} = \rho x(t),$$

where:

$x(t)$ represents magnitude of sleep inertia,

ρ is the constant rate of decline over time awake.

2. To preserve original model dynamics, a new prediction outcome variable was designated:

$$\frac{df(t)}{dt} = [v_1 + 2v_2 + \gamma x(t)]\frac{dp(t)}{dt} + \gamma p(t)\frac{dx(t)}{dt},$$

where: v_1 and v_2 are coefficients for relating lapses to effectiveness,

γ scalar for sleep inertia,

$x(t)$ magnitude of the sleep inertia effect (proportional to $p(t)$),

$p(t)$ performance impairment at time t.

Enhanced System of ODE’s

- Taken together, the system of ODE’s are as follows:

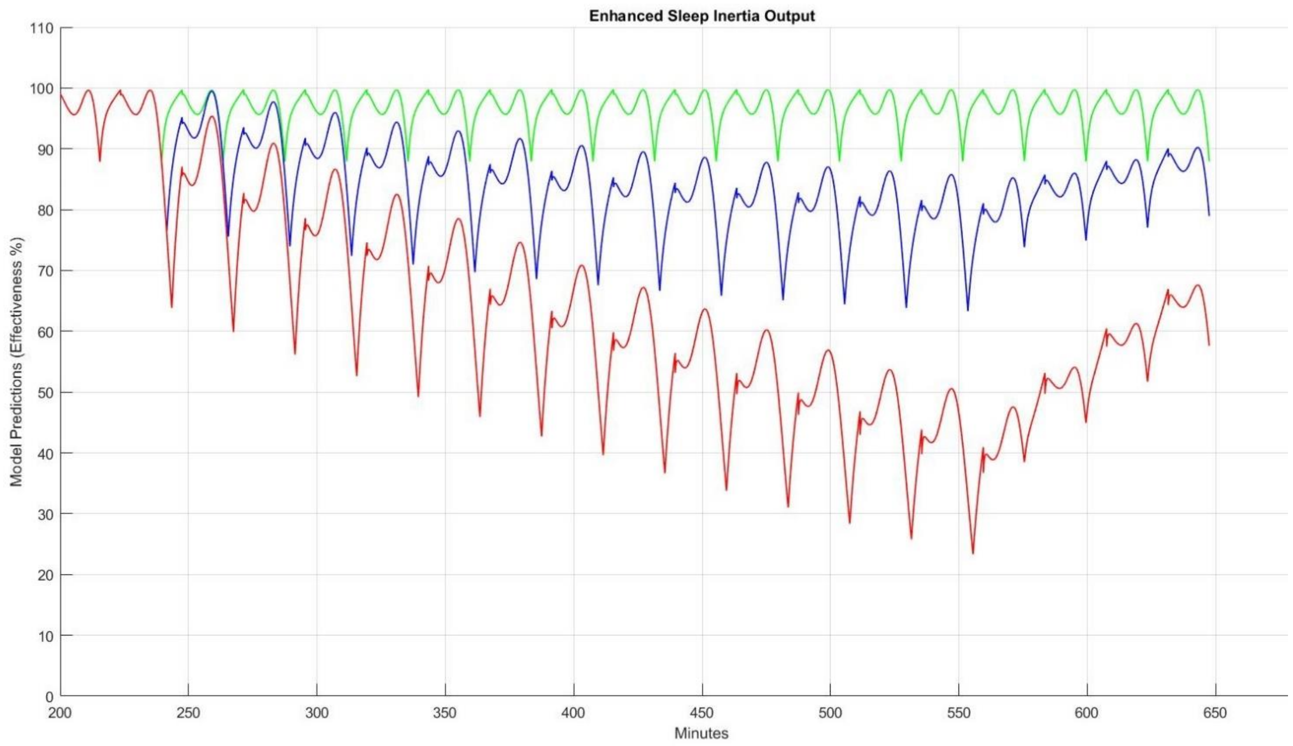
$$\frac{d\boldsymbol{p}(\boldsymbol{t})}{dt} = \boldsymbol{\alpha}_W[\mathbf{p}(\mathbf{t}) + \boldsymbol{\beta}_W\boldsymbol{u}(\boldsymbol{t})] + \boldsymbol{\kappa}(\boldsymbol{t})[\boldsymbol{c}(\boldsymbol{t}) + \boldsymbol{\mu}_W],$$

$$\frac{d\boldsymbol{u}(\boldsymbol{t})}{dt} = \boldsymbol{\eta}_W\boldsymbol{u}(\boldsymbol{t}),$$

$$\frac{d\boldsymbol{\kappa}(\boldsymbol{t})}{dt} = \boldsymbol{\lambda}_W\boldsymbol{\kappa}(\boldsymbol{t})\left(\mathbf{1} - \frac{\boldsymbol{\kappa}(\boldsymbol{t})}{\boldsymbol{\xi}}\right),$$

$$\frac{d\boldsymbol{x}(\boldsymbol{t})}{dt} = \boldsymbol{\gamma}\boldsymbol{x}(\boldsymbol{t}),$$

$$\frac{d\boldsymbol{f}(\boldsymbol{t})}{dt} = [\boldsymbol{v}_1 + \mathbf{2}\boldsymbol{v}_2 + \boldsymbol{\gamma}\boldsymbol{x}(\boldsymbol{t})]\frac{d\boldsymbol{p}(\boldsymbol{t})}{dt} + \boldsymbol{\gamma}\boldsymbol{p}(\boldsymbol{t})\frac{d\boldsymbol{x}(\boldsymbol{t})}{dt}.$$



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Conclusion

- ODE framework allows for additional mediators impacting the dynamics of sleep/wake regulation to be incorporated.
- Flexibility of ODE approach allows for additions both mediating fundamental dynamics, as well as, moderators causing transient changes.
- Enhanced model is conceptual and must be calibrated using actual data from investigations of sleep inertia.
- Further measurements of sleep inertia under a variety of homeostatic, circadian and allostatic states is needed for this estimation and validation of parameters.

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Questions?

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